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Model of Analysis of Influence of Radiation Processing on Transport of Charge Carriers

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Abstract

In this paper, we introduce a model for analysis of the influence of radiation processing on transport of charge carriers in p-n-junctions, which were manufactured in multilayer structures. The model gives the possibility to take into account a larger quantity of factors in comparison with existing models in the literature. We also introduce an analytical approach to analyze the transport of charge carriers. The approach makes it possible to simultaneously take into account changes in the properties of materials in space and time, as well as the nonlinearity of the considered transport.

Keywords: Charge carriers transport, p-n-junction, Radiation processing, Model of processes, Analytical approach for modelling.

1 | Introduction

Creation of new types of microelectronic devices and optimization of existing ones is the basis for the creation of new integrated circuits [1–5]. For both creating new types of microelectronic devices and optimizing existing ones, it is necessary to analyze processes occurring both during the creation of devices and during their operation [1–5]. In this paper, we consider a multilayer structure, which consists of a substrate and an epitaxial layer (see *Fig. 1*). An impurity is inputted into the epitaxial layer using diffusion or ion implantation. At the optimal value of continuance of annealing time, the sharpness of the p-n junction increases [6–9]. At the same time, with the increasing sharpness, the homogeneity of distribution of the considered concentration in the enriched area also increases. A qualitative structure of the p-n junction is shown in *Fig. 2*. In this figure and further in the text, the following designations are introduced: φ_k is the contact potential difference; U (t) is the applied difference of potentials; L_n and $-L_p$ are the boundaries of the p-n-junction area. The main aim of the present paper is the analysis of transport of charge carriers in p-n junctions after radiation processing

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on materials of p-n junctions. An accompanying aim of the paper is the introduction of an analytical approach for analysis of the considered transport, taking into account the simultaneous changes of the properties of materials in space and time and at the same time accounting for the nonlinearity of the considered transport.

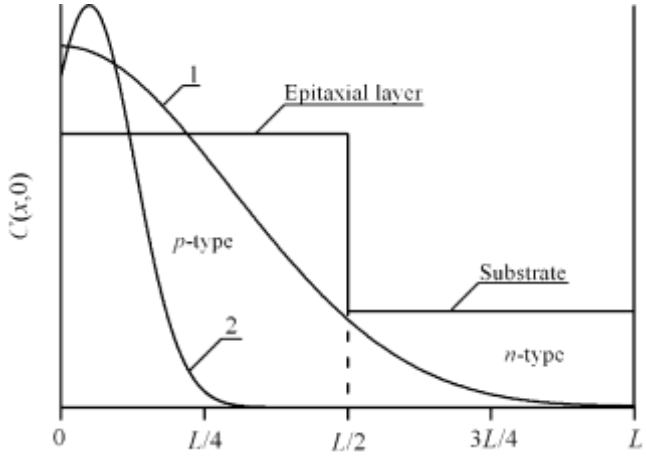


Fig. 1. Initial distributions of dopants and types of conductivity in the considered two-layer structure.

Curve 1 corresponds to the initial distribution of infused dopant. *Curve 2* corresponds to the initial distribution of implanted impurity. Initial distribution of dopant and types of conductivity of multiplayer structure

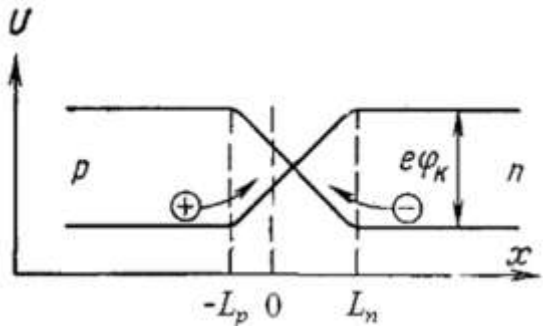


Fig. 2. Qualitative structure of p-n-junction.

2 | Method of Solution

Let us describe spatio-temporal distributions of concentrations of charge carriers in the considered p-n-junction by the following system of equations

$$\begin{cases} \dfrac{\partial n(x,t)}{\partial t} = G + \dfrac{\partial}{\partial x}\left[D_n \dfrac{\partial n(x,t)}{\partial x} \right] - \dfrac{\partial}{\partial x}\left\{ \mu_n n(x,t) \dfrac{\partial\left[\partial(x,t) + \partial_h(x,t)\right]}{\partial x} \right\} - \\ \qquad\qquad - k_{np}\left[n(x,t)p(x,t) - n_0 p_0 \right], \\ \dfrac{\partial p(x,t)}{\partial t} = G + \dfrac{\partial}{\partial x}\left[D_p \dfrac{\partial p(x,t)}{\partial x} \right] + \dfrac{\partial}{\partial x}\left\{ \mu_p p(x,t) \dfrac{\partial\left[\partial(x,t) + \partial_h(x,t)\right]}{\partial x} \right\} - \\ \qquad\qquad - k_{np}\left[n(x,t)p(x,t) - n_0 p_0 \right]. \end{cases} \quad (1)$$

Here ρ (x,t) (ρ = n, p) are the spatio-temporal distributions of concentrations of electrons (for ρ = n) and holes (for ρ = p); x is the coordinate; t is the time; ρ_0 are the equilibrium distributions of charge carriers; D_ρ are the diffusion coefficients of charge carriers; φ (x,t) is the distribution of potential in area of space charge; φ_h(x,t) is the potential barrier of heterojunction (now we consider, that we can neglect by quantum effects); G is the velocity of generation of electron-hole pairs; k_{np}(x, T) is the parameter of recombination; μ_ρ are the mobilities of charge carriers. It should be noted that the value of charge carrier mobility depends on the properties of the material and changes with variation of coordinate x in any multilayer structure. The value of the mobility also depends on the value of the tenseness of the electric field [4], [10]:

$$\mu_\rho(x)=\mu_{0\rho}\left\{1+\left[\frac{\mu_{0\rho}E(x)}{v_{sat}}\right]^{\beta_\rho}\right\}^{-\frac{1}{\beta_\rho}}, \tag{2}$$

where v_{sat} is the saturation value of drift velocity. Boundary and initial conditions for the system of *Eq. (1)* could be written as

$$-D_n\frac{\partial n(x,t)}{\partial x}\bigg|_{x=L_n}=J_{n0}(t),\ -D_n\frac{\partial n(x,t)}{\partial x}\bigg|_{x=-L_p}=J_{n0}(t),\ -D_p\frac{\partial p(x,t)}{\partial x}\bigg|_{x=L_n}=J_{p0}(t),,$$

$$-D_p\frac{\partial p(x,t)}{\partial x}\bigg|_{x=-L_p}=J_{p0}(t), n(x,0)=n_0(x), p(x,0)=p_0(x). \tag{3}$$

Let us determine the distribution of potential in the area of space charge as the solution of the following Poisson equation [4].

$$\frac{\partial^2\varphi(x,t)}{\partial x^2}=e\frac{N_a(x,t)-N_d(x,t)}{\varepsilon\varepsilon_0}, \tag{4}$$

where N_a (x,t) and N_d (x,t) are the spatiotemporal distributions of acceptor and donor dopants, respectively; e is the elementary charge; ε is the electric inductivity of materials; $\varepsilon_0\approx 8{,}85\cdot 10^{-12}$ F/m is the dielectric constant. Boundary conditions for the Poisson equation could be written as

$$\varphi(L_n,t)=\varphi_k+U(t),\ \varphi(-L_p,t)=U(t). \tag{5}$$

Further, let us consider single-carrier ionization, when N_a (x,t) = p (x,t) and N_d (x,t) = n (x,t). In this situation

$$\frac{\partial^2\varphi(x,t)}{\partial x^2}=e\frac{p(x,t)-n(x,t)}{\varepsilon\varepsilon_0} \tag{6}$$

One can obtain a solution of the above equation with boundary *Conditions (5)* in the following form

$$\varphi(x,t)=\varphi_k+U(t)+\frac{x-L_n}{L_p+L_n}\left\{\varphi_k+U(t)-\frac{e}{\varepsilon\varepsilon_0}\int_{-L_p}^{L_n}(L_n-v)\left[p(v,t)-n(v,t)\right]dv\right\}-$$

$$-\frac{e}{\varepsilon\varepsilon_0}\int_{x}^{L_n}(L_n-v)\left[p(v,t)-n(v,t)\right]dv. \tag{7}$$

Let us solve the system of *Eq. (1)* by the method of averaging of functional correction [9], [11]. In the framework of the method, we replace the concentrations of charge carriers n (x,t) and p (x,t) with their average

values α_{n1} and α_{p1}, which are not yet known. The replacement gives us the possibility to obtain first-order approximations of the concentrations in the following form

$$\begin{cases} \dfrac{\partial\, n_1(x,t)}{\partial\, t} = G - \alpha_{n1}\dfrac{\partial}{\partial\, x}\left\{\mu_n \dfrac{\partial\left[\varphi(x,t)+\varphi_h(x,t)\right]}{\partial\, x}\right\} - k_{np}\left(\alpha_{n1}\alpha_{p1} - n_0 p_0\right), \\ \dfrac{\partial\, p_1(x,t)}{\partial\, t} = G + \alpha_{p1}\dfrac{\partial}{\partial\, x}\left\{\mu_p \dfrac{\partial\left[\varphi(x,t)+\varphi_h(x,t)\right]}{\partial\, x}\right\} - k_{np}\left(\alpha_{n1}\alpha_{p1} - n_0 p_0\right). \end{cases} \quad (8)$$

Integration of the left and right parts of *Eq. (7)* on time leads to relations, which describe the first-order approximations of the required concentrations of charge carriers

$$\begin{cases} n_1(x,t) = \int_0^t G\, d\tau - \alpha_{n1}\dfrac{\partial}{\partial\, x}\int_0^t \mu_n \dfrac{\partial\left[\varphi(x,\tau)+\varphi_h(x,\tau)\right]}{\partial\, x} d\tau - \\ \qquad\qquad - \left(\alpha_{n1}\alpha_{p1} - n_0 p_0\right)\int_0^t k_{np}\, d\tau + n_0(x), \\ p_1(x,t) = \int_0^t G\, d\tau + \alpha_{p1}\dfrac{\partial}{\partial\, x}\int_0^t \mu_p \dfrac{\partial\left[\varphi(x,\tau)+\varphi_h(x,\tau)\right]}{\partial\, x} d\tau - \\ \qquad\qquad - \left(\alpha_{n1}\alpha_{p1} - n_0 p_0\right)\int_0^t k_{np}\, d\tau + p_0(x). \end{cases} \quad (9)$$

Not yet known average values α_{n1} and α_{p1} were determined by a framework standard iterative procedure of the method of averaging of function corrections, i.e.

$$\alpha_{\rho 2} = \frac{1}{\Theta\left(L_n + L_p\right)}\int_0^{\Theta}\int_{-L_p}^{L_n} \rho_1(x,t)\, dx\, dt. \quad (10)$$

Substitution of the first-order approximations of charge carriers into *Relation (9)* gives a possibility to obtain equations for calculations of not yet known average values α_{n1} and α_{p1}

$$\begin{cases} \alpha_{n1} = \int_0^{\Theta}\dfrac{\Theta - t}{\Theta\left(L_n + L_p\right)}\int_{-L_p}^{L_n} G\, dx\, dt - \dfrac{\alpha_{n1}\alpha_{p1} - n_0 p_0}{\Theta\left(L_n + L_p\right)}\int_0^{\Theta}(\Theta - t)\int_{-L_p}^{L_n} k_{np}\, dx\, dt \\ + \dfrac{1}{\Theta\left(L_n + L_p\right)}\int_0^{\Theta}\int_{-L_p}^{L_n} n_0(x)\, dx\, dt - \int_0^{\Theta}\mu_n \dfrac{U(t)(\Theta - t)}{\Theta\left(L_n + L_p\right)} dt \\ \alpha_{p1} = \int_0^{\Theta}\dfrac{\Theta - t}{\Theta\left(L_n + L_p\right)}\int_{-L_p}^{L_n} G\, dx\, dt - \dfrac{\alpha_{n1}\alpha_{p1} - n_0 p_0}{\Theta\left(L_n + L_p\right)}\int_0^{\Theta}(\Theta - t)\int_{-L_p}^{L_n} k_{np}\, dx\, dt \\ + \dfrac{1}{\Theta\left(L_n + L_p\right)}\int_0^{\Theta}\int_{-L_p}^{L_n} p_0(x)\, dx\, dt - \int_0^{\Theta}\mu_p \dfrac{U(t)(\Theta - t)}{\Theta\left(L_n + L_p\right)} dt. \end{cases} \quad (11)$$

Solution of the system of *Eq. (10)* by standard approaches [12] leads to the following result

$$\alpha_{p1} = \frac{b \pm \sqrt{b^2 - 4a\cdot c}}{2a}, \quad \alpha_{n1} = \left[\int_0^{\Theta}\frac{\Theta - t}{\Theta\left(L_n + L_p\right)}\int_{-L_p}^{L_n} G\, dx\, dt + \frac{n_0 p_0}{\Theta\left(L_n + L_p\right)}\int_0^{\Theta}(\Theta - t)\times \right.$$
$$\left. \int_{-L_p}^{L_n} k_{np}\, dx\, dt - \int_0^{\Theta}\mu_n \frac{U(t)(\Theta - t)}{\Theta\left(L_n + L_p\right)} dt + \frac{1}{\Theta\left(L_n + L_p\right)}\int_0^{\Theta}\int_{-L_p}^{L_n} n_0(x)\, dx\, dt \right] \times \quad (12)$$

$$\left[1+\frac{\alpha_{p1}}{\Theta\left(L_n+L_p\right)}\int_0^{\Theta}\left(\Theta-t\right)\int_{-L_p}^{L_n}k_{np}\,dx\,dt\right]^{-1}$$

where

$$a=\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}\,dx\,dt,\ b=1+\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}\,dx\,dt\left[\int_0^{\Theta}\mu_n\frac{U(t)(\Theta-t)}{\Theta\left(L_n+L_p\right)}dt-\right. \tag{13}$$

$$\left.\frac{n_0p_0}{\Theta\left(L_n+L_p\right)}\int_0^{\Theta}\left(\Theta-t\right)\int_{-L_p}^{L_n}k_{np}\,dx\,dt-\frac{1}{\Theta\left(L_n+L_p\right)}\int_0^{\Theta}\int_{-L_p}^{L_n}n_0(x)dx\,dt+\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}G\,dx\,dt\right]+$$

$$\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}\,dx\,dt\left[\frac{1}{\Theta}\int_0^{\Theta}\frac{\Theta-t}{L_n+L_p}\int_{-L_p}^{L_n}G\,dx\,dt+\frac{n_0p_0}{\Theta}\int_0^{\Theta}\frac{\Theta-t}{L_n+L_p}\int_{-L_p}^{L_n}k_{np}\,dx\,dt+\frac{1}{\Theta\left(L_n+L_p\right)}\times\right.$$

$$\left.\int_0^{\Theta}\int_{-L_p}^{L_n}p_0(x)dx\,dt-\int_0^{\Theta}\mu_p\frac{U(t)(\Theta-t)}{\Theta\left(L_n+L_p\right)}dt\right],\ c=\int_0^{\Theta}\mu_p\frac{U(t)(\Theta-t)}{\Theta\left(L_n+L_p\right)}dt-\frac{n_0p_0}{\Theta}\int_0^{\Theta}\frac{\Theta-t}{L_n+L_p}\int_{-L_p}^{L_n}k_{np}\,dx\,dt$$

$$-\frac{1}{\Theta}\int_0^{\Theta}\frac{\Theta-t}{\left(L_n+L_p\right)}\int_{-L_p}^{L_n}G\,dx\,dt-\frac{1}{\Theta\left(L_n+L_p\right)}\int_0^{\Theta}\int_{-L_p}^{L_n}p_0(x)dx\,dt.$$

Now we determine the second-order approximations of concentrations of charge carriers by using the standard iterative procedure of the method of averaging of function corrections [9], [11]. In the framework of the procedure, one shall replace the required concentrations in the following sums $n(x,t)\to\alpha_{n2}+n_1(x,t)$ and $p(x,t)\to\alpha_{p2}+p_1(x,t)$ in the right sides of *Eq. (1)*. The replacement gives a possibility to obtain an equation for calculation of the second-order approximations of concentrations of charge carriers in the following form

$$\begin{cases}\dfrac{\partial n_2(x,t)}{\partial t}=\dfrac{\partial}{\partial \overleftrightarrow{x}}\left[D_n\dfrac{\partial n_1(x,t)}{\partial x}\right]-\dfrac{\partial}{\partial \overleftrightarrow{x}}\left\{\mu_n[\alpha_{n2}+n_1(x,t)]\dfrac{\partial[\phi(x,t)+\phi_h(x,t)]}{\partial x}\right\}-\\ k_{np}\{[\alpha_{n2}+n_1(x,t)][\alpha_{p2}+p_1(x,t)]-n_0p_0\}+G\\ \dfrac{\partial p_2(x,t)}{\partial t}=\dfrac{\partial}{\partial \overleftrightarrow{x}}\left[D_p\dfrac{\partial p_1(x,t)}{\partial x}\right]+\dfrac{\partial}{\partial \overleftrightarrow{x}}\left\{\mu_p[\alpha_{p2}+p_1(x,t)]\dfrac{\partial[\phi(x,t)+\phi_h(x,t)]}{\partial x}\right\}-\\ k_{np}\{[\alpha_{n2}+n_1(x,t)][\alpha_{p2}+p_1(x,t)]-n_0p_0\}+G.\end{cases} \tag{14}$$

Integration of the left and right parts of *Eq. (13)* on time leads to relations, which describe the second-order approximations of the required concentrations of charge carriers

$$\begin{cases}n_2(x,t)=n_0(x)+\displaystyle\int_0^t G\,d\tau-\frac{\partial}{\partial \overleftrightarrow{x}}\left\{\int_0^t\mu_n[\alpha_{n2}+n_1(x,\tau)]\frac{\partial[\phi(x,\tau)+\phi_h(x,\tau)]}{\partial x}d\tau\right\}-\\ \displaystyle\int_0^t k_{np}\{[\alpha_{n2}+n_1(x,\tau)][\alpha_{p2}+p_1(x,\tau)]-n_0p_0\}d\tau+\frac{\partial}{\partial \overleftrightarrow{x}}\int_0^t D_n\frac{\partial n_1(x,\tau)}{\partial x}d\tau\\ p_2(x,t)=p_0(x)+\displaystyle\int_0^t G\,d\tau+\frac{\partial}{\partial \overleftrightarrow{x}}\left\{\int_0^t\mu_p[\alpha_{n2}+p_1(x,\tau)]\frac{\partial[\phi(x,\tau)+\phi_h(x,\tau)]}{\partial x}d\tau\right\}-\\ \displaystyle\int_0^t k_{np}\{[\alpha_{n2}+n_1(x,\tau)][\alpha_{p2}+p_1(x,\tau)]-n_0p_0\}d\tau+\frac{\partial}{\partial \overleftrightarrow{x}}\int_0^t D_p\frac{\partial p_1(x,\tau)}{\partial x}d\tau.\end{cases} \tag{15}$$

Not yet known average values α_{n2} and α_{p2} were determined by the framework standard iteration procedure of the method of averaging of function corrections, i.e.

$$\alpha_{\rho 2}=\frac{1}{\Theta\left(L_n+L_p\right)}\int_0^{\Theta}\int_{-L_p}^{L_n}\left[\rho_2(x,t)-\rho_1(x,t)\right]dx\,dt, \tag{16}$$

where $\rho = n,p$; Θ is the continuation of observation of dynamics of charge carriers. Substitution of the first- and the second-order approximations in *Relation (15)* gives a possibility to obtain the following system of equations to determine average values α_{n2} and α_{p2}.

$$\begin{cases}\alpha_{n2}=\dfrac{\alpha_{n1}\alpha_{p1}-\alpha_{n2}\alpha_{p2}}{\Theta\left(L_n+L_p\right)}\displaystyle\int_0^{\Theta}(\Theta-t)\int_{-L_p}^{L_n}k_{np}\,dx\,dt-\frac{\alpha_{n2}}{\Theta}\int_0^{\Theta}\frac{\Theta-t}{L_n+L_p}\int_{-L_p}^{L_n}k_{np}p_1(x,t)\,dx\,dt-\\ \dfrac{\alpha_{p2}}{\Theta}\displaystyle\int_0^{\Theta}\frac{\Theta-t}{L_n+L_p}\int_{-L_p}^{L_n}k_{np}n_1(x,t)\,dx\,dt-\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}n_1(x,t)p_1(x,t)\,dx\,dt-\\ \left(\alpha_{n2}-\alpha_{n1}\right)\displaystyle\int_0^{\Theta}(\Theta-t)v_{sat}(t)\,dt\\ \alpha_{p2}=\dfrac{\alpha_{n1}\alpha_{p1}-\alpha_{n2}\alpha_{p2}}{\Theta\left(L_n+L_p\right)}\displaystyle\int_0^{\Theta}(\Theta-t)\int_{-L_p}^{L_n}k_{np}\,dx\,dt-\frac{\alpha_{n2}}{\Theta}\int_0^{\Theta}\frac{\Theta-t}{L_n+L_p}\int_{-L_p}^{L_n}k_{np}p_1(x,t)\,dx\,dt-\\ \dfrac{\alpha_{p2}}{\Theta}\displaystyle\int_0^{\Theta}\frac{\Theta-t}{L_n+L_p}\int_{-L_p}^{L_n}k_{np}n_1(x,t)\,dx\,dt-\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}n_1(x,t)p_1(x,t)\,dx\,dt-\\ \left(\alpha_{p2}-\alpha_{p1}\right)\displaystyle\int_0^{\Theta}(\Theta-t)v_{sat}(t)\,dt.\end{cases} \tag{17}$$

The solution of the above equation could be written as

$$\alpha_{p2}=\frac{-e\pm\sqrt{e^2-4df}}{2d}.$$

$$\alpha_{n2}=\left[\frac{\alpha_{n1}\alpha_{p1}}{\Theta\left(L_n+L_p\right)}\int_0^{\Theta}(\Theta-t)\int_{-L_p}^{L_n}k_{np}\,dx\,dt-\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}n_1(x,t)p_1(x,t)\,dx\,dt+\right.$$
$$\left.\alpha_{n1}\int_0^{\Theta}(\Theta-t)v_{sat}(t)\,dt-\alpha_{p2}\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}n_1(x,t)\,dx\,dt\right]\Bigg/\left[1+\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\times\right. \tag{18}$$
$$\left.\times\int_{-L_p}^{L_n}k_{np}p_1(x,t)\,dx\,dt+\int_0^{\Theta}(\Theta-t)v_{sat}(t)\,dt+\alpha_{p2}\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}\,dx\,dt\right],$$

where

$$d=\left[1+2\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}n_1(x,t)\,dx\,dt+\int_0^{\Theta}(\Theta-t)v_{sat}(t)\,dt\right]\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}\,dx\,dt;$$

$$e=\left\{\left[\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}n_1(x,t)p_1(x,t)\,dx\,dt-\alpha_{p1}\int_0^{\Theta}(\Theta-t)v_{sat}(t)\,dt-\alpha_{n1}\alpha_{p1}\int_0^{\Theta}\int_{-L_p}^{L_n}k_{np}\,dx\,dt\right]\times\right.$$
$$\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}\,dx\,dt+\left[1+\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}n_1(x,t)\,dx\,dt+\int_0^{\Theta}(\Theta-t)v_{sat}(t)\,dt\right]\times$$

$$\left[1+\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}p_1(x,t)dxdt+\int_0^{\Theta}(\Theta-t)v_{sat}(t)dt\right]+\int_0^{\Theta}\int_{-L_p}^{L_n}k_{np}n_1(x,t)dx\times$$

$$\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}dt\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}p_1(x,t)dxdt+\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}dxdt\left[\frac{\alpha_{n1}\alpha_{p1}}{\Theta\left(L_n+L_p\right)}\times\right.$$

$$\left.\int_0^{\Theta}(\Theta-t)\int_{-L_p}^{L_n}k_{np}dxdt-\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}n_1(x,t)p_1(x,t)dxdt+\alpha_{n1}\int_0^{\Theta}(\Theta-t)\,v_{sat}(t)\,dt\right]\Bigg\};$$

$$f=\left[\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}n_1(x,t)p_1(x,t)dxdt-\alpha_{p1}\int_0^{\Theta}(\Theta-t)v_{sat}(t)dt-\alpha_{n1}\alpha_{p1}\int_0^{\Theta}\int_{-L_p}^{L_n}k_{np}dxdt\right]\times$$

$$\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}p_1(x,t)dxdt\left[\int_0^{\Theta}(\Theta-t)\,v_{sat}(t)dt\,+1+\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}p_1(x,t)dxdt\right]\times$$

$$\left[\frac{\alpha_{n1}\alpha_{p1}}{\Theta\left(L_n+L_p\right)}\int_0^{\Theta}(\Theta-t)\int_{-L_p}^{L_n}k_{np}dxdt-\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}n_1(x,t)p_1(x,t)dxdt+\alpha_{n1}\int_0^{\Theta}(\Theta-t)\times\right.$$

$$\left.v_{sat}(t)dt\right]-\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}p_1(x,t)dxdt\left[\alpha_{n1}\int_0^{\Theta}(\Theta-t)v_{sat}(t)dt-\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}\times\right.$$

$$\left.n_1(x,t)p_1(x,t)dxdt+\frac{\alpha_{n1}\alpha_{p1}}{\Theta}\int_0^{\Theta}\frac{\Theta-t}{L_n+L_p}\int_{-L_p}^{L_n}k_{np}dxdt\right]-\int_0^{\Theta}\frac{\Theta-t}{L_n+L_p}\int_{-L_p}^{L_n}\frac{k_{np}}{\Theta}p_1(x,t)dxdt\times$$

$$\left[\frac{\alpha_{n1}\alpha_{p1}}{\Theta}\int_0^{\Theta}\frac{\Theta-t}{L_n+L_p}\int_{-L_p}^{L_n}k_{np}dxdt-\frac{1}{\Theta}\int_0^{\Theta}\frac{\Theta-t}{L_n+L_p}\int_{-L_p}^{L_n}k_{np}n_1(x,t)\,p_1(x,t)dxdt+\alpha_{n1}\int_0^{\Theta}(\Theta-t)\times\right.$$

$$\left.v_{sat}(t)dt\right]-\int_0^{\Theta}\frac{\Theta-t}{\Theta\left(L_n+L_p\right)}\int_{-L_p}^{L_n}k_{np}p_1(x,t)dxdt\left[\frac{\alpha_{n1}\alpha_{p1}}{\Theta}\int_0^{\Theta}\frac{\Theta-t}{L_n+L_p}\int_{-L_p}^{L_n}k_{np}dxdt-\int_0^{\Theta}\frac{\Theta-t}{L_n+L_p}\times\right.$$

$$\left.\frac{1}{\Theta}\int_{-L_p}^{L_n}k_{np}n_1(x,t)p_1(x,t)dxdt+\alpha_{n1}\int_0^{\Theta}(\Theta-t)v_{sat}(t)dt\right].$$

Now let us calculate spatio-temporal distributions of concentrations of radiation defects. We calculate the above distributions by solving the following system of equations [13], [14].

$$\frac{\partial I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial y}\right]-$$

$$k_{I,I}(x,y,z,T)I^2(x,y,z,t)+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial z}\right]-k_{I,V}(x,y,z,T)\times \quad (19)$$

$$\times I(x,y,z,t)V(x,y,z,t).$$

$$\frac{\partial V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial y}\right]-$$

$$k_{V,V}(x,y,z,T)V^2(x,y,z,t)+\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial z}\right]-k_{I,V}(x,y,z,T)\times \quad (20)$$

$$\times I(x,y,z,t)V(x,y,z,t),$$

with boundary and initial conditions

$$\left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=0}=0,$$

$$\left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

$$\left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=0}=0,$$

$$\left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

$$I(x,y,z,0)=f_I(x,y,z),\ V(x,y,z,0)=f_V(x,y,z).$$

Here I (x,y,z,t) is the spatio-temporal distribution of radiation interstitials with equilibrium distribution I^*; V (x,y,z,t) is the spatio-temporal distribution of radiation vacancies with equilibrium distribution V^*; $D_I(x,y,z,T)$ and $D_V(x,y,z,T)$ are the diffusion coefficients of interstitials and vacancies, respectively; terms $V^2(x,y,z,t)$ and $I^2(x,y,z,t)$ correspond to generation of divacancies and diinterstitials (see, for example, [15] and appropriate references in the work); $k_{I,V}(x,y,z,T)$ is the parameters of recombination of point radiation defects; $k_{I,I}(x,y,z,T)$ and $k_{V,V}(x,y,z,T)$ are the parameters of generation of complexes of simplest complexes of point radiation defects (diinterstitials and divacancies). We calculate the spatio-temporal distributions of concentrations of divacancies Φ_V (x,y,z,t) and diinterstitials Φ_I (x,y,z,t) as solutions of the following system of equations [14–16]

$$\frac{\partial \Phi_I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right]+$$

$$\frac{\partial}{\partial z}\left[D_{\Phi I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right]+k_I(x,y,z,T)I(x,y,z,t)+k_{I,I}(x,y,z,T)\times$$

$$I^2(x,y,z,t). \quad (21)$$

$$\frac{\partial \Phi_V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right]+$$

$$\frac{\partial}{\partial z}\left[D_{\Phi V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right]+k_V(x,y,z,T)V(x,y,z,t)+k_{V,V}(x,y,z,T)\times$$

$$\times V^2(x,y,z,t).$$

with boundary and initial conditions

$$\left.\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right|_{x=0}=0,\quad \left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\quad \left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=0}=0,\quad \left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,$$
$$\left.\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right|_{z=0}=0,\quad \left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,\quad \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right|_{x=0}=0,\quad \left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,$$
$$\left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=0}=0,\quad \left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\quad \left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=0}=0,\quad \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,\quad \Phi_I \tag{22}$$
$$(x,y,z,0)=f_{\Phi I}(x,y,z),\ \Phi_V(x,y,z,0)=f_{\Phi V}(x,y,z).$$

Here $D_{\Phi I}(x,y,z,T)$ and $D_{\Phi V}(x,y,z,T)$ are the diffusion coefficients of diinterstitials and divacancies, respectively; $k_I(x,y,z,T)$ and $k_V(x,y,z,T)$ are the parameters of decay of simplest complexes of defects.

To calculate the distributions, we also used the method of averaging of function corrections. In the framework of the approach, we replace the concentrations I (x,y,z,t) and V (x,y,z,t) with their average values α_{1I} and α_{1V}. After the replacement, we obtained equations for the first-order approximations of concentrations of point radiation defects I_1 (x,y,z,t) and V_1 (x,y,z,t)

$$\frac{\partial I_1(x,y,z,t)}{\partial t}=-\alpha_{1I}^2 k_{I,I}(x,y,z,T)-\alpha_{1I}\alpha_{1V}k_{I,V}(x,T).$$
$$\frac{\partial V_1(x,y,z,t)}{\partial t}=-\alpha_{1V}^2 k_{V,V}(x,y,z,T)-\alpha_{1I}\alpha_{1V}k_{I,V}(x,T).$$

Integration on time of the left and right sides of the above equations gives us the possibility to obtain the functions I_1 (x,y,z,t) and V_1 (x,y,z,t) in the final form

$$I_1(x,y,z,t)=f_I(x,y,z)-\alpha_{1I}^2\int_0^t k_{I,I}(x,y,z,T)\,d\tau-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(x,y,z,T)\,d\tau.$$
$$V_1(x,y,z,t)=f_V(x,y,z)-\alpha_{1V}^2\int_0^t k_{V,V}(x,y,z,T)\,d\tau-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(x,y,z,T)\,d\tau. \tag{23}$$

Average values of the first-order approximations of concentrations of point radiation defects could be calculated by the standard relation

$$\alpha_{1R}=\frac{1}{L_xL_yL_z\Theta}\int_0^{\Theta}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}R_1(x,y,z,t)\,dz\,dy\,dx\,dt, \tag{24}$$

where R = I,V. Substitution of Solutions *Eq. (23)* into the above relation gives us the possibility to obtain the following equations for the average values α_{1I} and α_{1V}

$$\begin{cases}\alpha_{1I}B_{I,I}^{1010}+\alpha_{1V}B_{I,V}^{1010}=\Theta C_I-L_xL_yL_z\Theta,\\ \alpha_{1I}B_{I,V}^{1010}+\alpha_{1V}B_{V,V}^{1010}=\Theta C_V-L_xL_yL_z\Theta,\end{cases}$$

where $B_{R,R}^{ijkl}=\int_0^{\Theta}(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_{R,R}(x,y,z,T)I_i^j(x,y,z,t)V_k^l(x,y,z,t)\,dz\,dy\,dx\,dt$, $C_R=\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_R(x,y,z)\,dz\,dy\,dx$.

Solution of the above system of equations is

$$\alpha_{1V}=\left[B_{I,V}(L_xL_yL_z\Theta-C_V)-L_xL_yL_z\Theta B_{V,V}\right]\frac{B_{V,V}B_{I,I}+B_{I,V}^2}{B_{I,I}B_{V,V}},\ \alpha_{1I}=\frac{C_I-L_xL_yL_z\Theta-\alpha_{1V}B_{I,V}}{B_{I,I}}.$$

Approximations of the second and higher orders of concentrations of point radiation defects in the framework of the method of averaging of function corrections could be obtained by using a standard iteration procedure, i.e., by replacing the functions I (x, y,z,t) and V (x,y,z,t) in the following sums $\alpha_{nI}+I_{n-1}(x,y,z,t)$ and $\alpha_{nV}+V_{n-1}(x,y,z,t)$, where n is the order of approximation. In the present paper, we consider only the second-order approximations of the functions I (x,y,z,t) and V (x,y,z,t). The above substitution and integration on

time on the left and right sides give us the possibility to obtain the second-order approximations of concentrations of defects $I_2(x,y,z,t)$ and $V_2(x,y,z,t)$ in the following form

$$I_2(x,y,z,t)=\frac{\partial}{\partial x}\left[\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial x}d\tau\right]+\frac{\partial}{\partial y}\left[\int_0^t D_I(x,y,z,T)\times\right.$$
$$\left.\frac{\partial I_1(x,y,z,\tau)}{\partial y}d\tau\right]+\frac{\partial}{\partial z}\left[\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial z}d\tau\right]-\int_0^t k_{I,I}(x,y,z,T)\times$$
$$[\alpha_{2I}+I_1(x,y,z,\tau)]^2d\tau-\int_0^t k_{I,V}(x,T)[\alpha_{2I}+I_1(x,y,z,\tau)][\alpha_{2V}+V_1(x,y,z,\tau)]d\tau+$$
$$+f_I(x,y,z).$$

$$V_2(x,y,z,t)=\frac{\partial}{\partial x}\left[\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial x}d\tau\right]+\frac{\partial}{\partial y}\left[\int_0^t D_V(x,y,z,T)\times\right.$$
$$\left.\frac{\partial V_1(x,y,z,\tau)}{\partial y}d\tau\right]+\frac{\partial}{\partial z}\left[\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial z}d\tau\right]-\int_0^t k_{V,V}(x,y,z,T)\times$$
$$[\alpha_{2V}+V_1(x,y,z,\tau)]^2d\tau-\int_0^t k_{I,V}(x,T)[\alpha_{2I}+I_1(x,y,z,\tau)][\alpha_{2V}+V_1(x,y,z,\tau)]d\tau+$$
$$f_V(x,y,z).$$

Average values of the second-order approximation and approximations of the higher orders of concentrations of point radiation defects α_{nI} and α_{nV} could be calculated by using the standard relation [9], [11].

$$\alpha_{nR}=\frac{1}{L_xL_yL_z\Theta}\int_0^\Theta\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left[R_n(x,y,z,t)-R_{n-1}(x,y,z,t)\right]dz\,dy\,dx\,dt. \qquad (25)$$

Substitution of the first- and second-order approximations of the functions I (x,y,z,t) and V (x,y,z,t) into *Eq. (24)* gives us the possibility to obtain equations for the average values α_{2I} and α_{2V}

$$\alpha_{2I}L_xL_yL_z\Theta=C_I-\int_0^\Theta(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_{I,I}(x,y,z,T)\left[\alpha_{2I}+I_1(x,y,z,t)\right]^2dz\,dy\,dx\,dt-\alpha_{1I}\times$$
$$\Theta L_xL_yL_z-\int_0^\Theta(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_{I,V}(x,T)\left[\alpha_{2I}+I_1(x,y,z,t)\right]\left[\alpha_{2V}+V_1(x,y,z,t)\right]dz\,dy\,dx\,dt.$$

$$\alpha_{2V}L_xL_yL_z\Theta=C_V-\int_0^\Theta(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_{V,V}(x,y,z,T)\left[\alpha_{2V}+V_1(x,y,z,t)\right]^2dz\,dy\,dx\,dt-\alpha_{1V}\times$$
$$L_xL_yL_z\Theta-\int_0^\Theta(\Theta-t)\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_{I,V}(x,T)\left[\alpha_{2I}+I_1(x,y,z,t)\right]\left[\alpha_{2V}+V_1(x,y,z,t)\right]dz\,dy\,dx\,dt.$$

The solution of the above system of equations could be written as

$$\alpha_{2I}=\frac{1}{2}\sqrt{\frac{1}{4}\left(a_6+a_2/a_1\right)^2-4\left[y+\left(a_2y-a_4\right)/a_1a_6\right]}-\frac{1}{4}\left(a_6+a_2/a_1\right),$$
$$\alpha_{2V}=\left[C_I-\alpha_{2I}\left(2B_{I,I}^{1110}+L_xL_yL_z\Theta+B_{I,V}^{1011}\right)-B_{I,I}^{1210}-B_{I,V}^{1111}-\alpha_{1I}L_xL_yL_z\Theta-\alpha_{2I}^2B_{I,I}^{1010}\right]\times$$
$$\left(\alpha_{2I}B_{I,V}^{1010}+B_{I,V}^{1110}\right).$$

where

$$a_1 = B_{V,V}^{1010}\left(B_{I,I}^{1010}\right)^2 - B_{I,I}^{1010}\left(B_{I,V}^{1010}\right)^2,\ a_2 = B_{I,I}^{1010}B_{V,V}^{1010}\left(2B_{I,I}^{1110} + L_xL_yL_z\Theta + B_{I,V}^{1011}\right) - \left(L_x \times\right.$$

$$\left.L_yL_z\Theta + 2B_{I,I}^{1110} + B_{I,V}^{1011}\right)\left(B_{I,V}^{1010}\right)^2 - B_{I,V}^{1010}\left(2B_{V,V}^{1110} + L_xL_yL_z\Theta + B_{I,V}^{1011} + B_{I,V}^{1110}\right)B_{I,I}^{1010} + B_{I,V}^{1110},$$

$$a_3 = B_{V,V}^{1010}\left(2B_{I,I}^{1110} + L_xL_yL_z\Theta + B_{I,V}^{1011}\right)^2 + \left(B_{I,V}^{1010}\right)^2\left(C_I - B_{I,I}^{1210} - B_{I,V}^{1111} - \alpha_{1I}L_xL_yL_z\Theta\right) - \left(2B_{V,V}^{1110} +\right.$$

$$-B_{I,V}^{1010}\left(2B_{V,V}^{1110} + L_xL_yL_z\Theta + B_{I,V}^{1011} + B_{I,V}^{1110}\right)\left(2B_{I,I}^{1110} + L_xL_yL_z\Theta + B_{I,V}^{1011}\right) + B_{I,V}^{1111} + \alpha_{1V}L_xL_yL_z\Theta +$$

$$-B_{I,I}^{1010}B_{I,V}^{1110}\left(2B_{V,V}^{1110} + L_xL_yL_z\Theta + B_{I,V}^{1011}\right) + B_{V,V}^{1210} - C_V + 2B_{I,V}^{1110}B_{I,V}^{1010}B_{I,V}^{1110},$$

$$a_4 = B_{I,V}^{1010}\left(2B_{V,V}^{1110} +\right.$$

$$\left.B_{I,V}^{1011} + B_{I,V}^{1110} + L_xL_yL_z\Theta\right)\left(C_I - B_{I,I}^{1210} - B_{I,V}^{1111} - \alpha_{1I}L_xL_yL_z\Theta\right) - \left(2B_{I,I}^{1110} + L_xL_yL_z\Theta + B_{I,V}^{1011}\right) \times$$

$$B_{V,V}^{1010}\left(C_I - B_{I,I}^{1210} - B_{I,V}^{1111} - \alpha_{1I}L_xL_yL_z\Theta\right) - B_{V,V}^{1010}\left(2B_{I,I}^{1110} + L_xL_yL_z\Theta + B_{I,V}^{1011}\right)\left(C_I - B_{I,I}^{1210} -\right.$$

$$\left.B_{I,V}^{1111} - \alpha_{1I}L_xL_yL_z\Theta\right) - B_{I,V}^{1110}\left(2B_{I,I}^{1110} + L_xL_yL_z\Theta + B_{I,V}^{1011}\right)\left(2B_{V,V}^{1110} + L_xL_yL_z\Theta + B_{I,V}^{1011}\right) + B_{I,V}^{1110} \times$$

$$2B_{I,V}^{1010}\left(B_{I,V}^{1111} + \alpha_{1V}L_xL_yL_z\Theta + B_{V,V}^{1210} - C_V\right) + B_{I,V}^{1110}\left(B_{I,V}^{1110}\right)^2,$$

$$a_5 = B_{V,V}^{1010}\left(C_I - B_{I,I}^{1210} - B_{I,V}^{1111} -\right.$$

$$\left.\alpha_{1I}L_xL_yL_z\Theta\right)^2 + B_{V,V}^{1010}B_{I,I}^{1010}\left(\alpha_{1I}L_xL_yL_z\Theta + B_{I,I}^{1210} + B_{I,V}^{1111} - C_I\right) + \left(2B_{V,V}^{1110} + L_xL_yL_z\Theta + B_{I,V}^{1011}\right) \times$$

$$B_{I,V}^{1110}\left(C_I - B_{I,I}^{1210} - B_{I,V}^{1111} - \alpha_{1I}L_xL_yL_z\Theta\right) + \left(B_{I,V}^{1110}\right)^2\left(B_{I,V}^{1111} + \alpha_{1V}L_xL_yL_z\Theta + B_{V,V}^{1210} - C_V\right),$$

$$q = \frac{2a_3^3 + 27a_1^2a_4 - 9a_1a_3\left(a_2a_4 - 4a_5\right)}{432a_1^3},\ y = \sqrt[3]{-q + \sqrt{q^2 + p^3}} - \sqrt[3]{q + \sqrt{q^2 + p^3}},$$

$$p = \frac{24a_1a_3 - a_2^2}{13824a_1^2}.\ a_6 = -\sqrt{8y + \frac{a_2^2}{a_1^2} - 4\frac{a_3}{a_1}}.$$

The system of *Eq. (21)* is solved using the method of averaging of function corrections. In the framework of the approach, we replace the functions Φ_I (x,y,z,t) and Φ_V (x,y,z,t) on the right-hand side of *Eq. (21)* with their average values $\alpha_{1\Phi I}$ and $\alpha_{1\Phi V}$. After the replacement, we obtain the equations for the first-order approximations of concentrations of simplest complexes of radiation defects Φ_{1I} (x,y,z,t) and Φ_{1V} (x,y,z,t) in the following form

$$\frac{\partial\Phi_{I1}(x,y,z,t)}{\partial t} = k_{I,I}(x,y,z,T)I^2(x,y,z,t) - k_I(x,y,z,T)I(x,y,z,t).$$

$$\frac{\partial\Phi_{V1}(x,y,z,t)}{\partial t} = k_{V,V}(x,y,z,T)V^2(x,y,z,t) - k_V(x,y,z,T)V(x,y,z,t).$$

Integration of the left and right parts of the equations on time gives us the possibility to obtain the first-order approximations of concentrations of divacancies and diinterstitials in the final form

$$\Phi_{I1}(x,y,z,t) = k_{I,I}(x,y,z,T)\,I^2(x,y,z,t) + f_{\Phi I}(x,y,z) - k_I(x,y,z,T)\,I(x,y,z,t).$$

$$\Phi_{V1}(x,y,z,t) = k_{V,V}(x,y,z,T)\,V^2(x,y,z,t) + f_{\Phi V}(x,y,z) - k_V(x,y,z,T)\,V(x,y,z,t). \quad \textbf{(26)}$$

Average values $\alpha_{1\Phi I}$ and $\alpha_{1\Phi V}$ of the first-order approximations of concentrations of divacancies and diinterstitials could be calculated by standard relations, which are analogous to *Relations (23)*. By calculation, we obtain the following result

$$\alpha_{1\Phi I} = \left(\Theta F_I - E_I^1 + E_I^2\right)/L_xL_yL_z\Theta,\ \alpha_{1\Phi V} = \left(\Theta F_V - E_V^1 + E_V^2\right)/L_xL_yL_z\Theta,$$

where,

$$E_R^1 = \int_0^\Theta (\Theta - t) \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} k_R(x,y,z,t) R(x,y,z,T) dz dy dx dt,\ E_R^2 = \int_0^\Theta (\Theta - t) \times$$

$$\int_0^{L_x} \int_0^{L_y} \int_0^{L_z} k_{R,R}(x,y,z,T) R^2(x,y,z,t) dz dy dx dt,\ F_R = \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} f_{\Phi R}(x,y,z) dz dy dx.$$

The second-order approximations of concentrations of divacancies and diinterstitials could be calculated by the standard iteration procedure of the method of averaging of function corrections. The approximations could be written as

$$\Phi_{I2}(x,y,z,t) = \frac{\partial}{\partial x}\left[\int_0^t D_{\Phi I}(x,y,z,T) \frac{\partial \Phi_{I1}(x,y,z,\tau)}{\partial x} d\tau\right] + \frac{\partial}{\partial y}\left[\int_0^t \frac{\partial \Phi_{I1}(x,y,z,\tau)}{\partial y} \times\right.$$

$$\left. D_{\Phi I}(x,y,z,T) d\tau\right] + \frac{\partial}{\partial z}\left[\int_0^t D_{\Phi I}(x,y,z,T) \frac{\partial \Phi_{I1}(x,y,z,\tau)}{\partial z} d\tau\right] + \int_0^t k_I(x,y,z,T) \times$$

$$I(x,y,z,\tau) d\tau + \int_0^t k_{I,I}(x,y,z,T) I^2(x,y,z,\tau) d\tau + f_{\Phi I}(x,y,z).$$

$$\Phi_{V2}(x,y,z,t) = \frac{\partial}{\partial x}\left[\int_0^t D_{\Phi V}(x,y,z,T) \frac{\partial \Phi_{V1}(x,y,z,\tau)}{\partial x} d\tau\right] + \frac{\partial}{\partial y}\left[\int_0^t \frac{\partial \Phi_{V1}(x,y,z,\tau)}{\partial y} \times\right.$$

$$\left. D_{\Phi V}(x,y,z,T) d\tau\right] + \frac{\partial}{\partial z}\left[\int_0^t D_{\Phi V}(x,y,z,T) \frac{\partial \Phi_{V1}(x,y,z,\tau)}{\partial z} d\tau\right] + \int_0^t k_V(x,y,z,T) \times$$

$$V(x,y,z,\tau) d\tau + \int_0^t k_{V,V}(x,y,z,T) V^2(x,y,z,\tau) d\tau + f_{\Phi V}(x,y,z).$$

Average values $\alpha_{2\Phi I}$ and $\alpha_{2\Phi V}$ could be determined by the standard relation. The relation is similar to relation *(23)*. Results of calculation of the average values could be written as

$$\alpha_{2\Phi I} = \frac{G_I^1 + F_{RI} + E_I^2 + E_I^1 - \alpha_{1\Phi I} L_x L_y L_z \Theta}{L_x L_y L_z \Theta},\ \alpha_{2\Phi V} = \frac{G_V^1 + F_{RV} + E_V^2 + E_V^1 - \alpha_{1\Phi V} L_x L_y L_z \Theta}{L_x L_y L_z \Theta},$$

where,

$$G_R^i = \int_0^\Theta (\Theta - t) \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \left\{\frac{\partial}{\partial x}\left[D_{\Phi R}(x,y,z,T) \frac{\partial \Phi_{Ri}(x,y,z,t)}{\partial x}\right] + \frac{\partial}{\partial y}[D_{\Phi R}(x,y,z,T) + \right.$$

$$\left. \times \frac{\partial \Phi_{Ri}(x,y,z,t)}{\partial y}\right] + \frac{\partial}{\partial z}\left[D_{\Phi R}(x,y,z,T) \frac{\partial \Phi_{Ri}(x,y,z,t)}{\partial z}\right]\right\} dz dy dx dt.$$

Analysis of spatio-temporal distributions of concentrations of charge carriers and radiation defects has been done analytically by considering their second-order approximations. Analytical results were checked by comparison with results of direct numerical simulation.

3 | Discussion

In this paper, we analyzed the solution of *Eq. (1)* with account solutions of *Eqs. (18)* and *(20)*. We obtained that increasing the intensity of radiation processing of materials leads to a decrease in charge carriers mobility (see *Fig. 1*). The result has qualitative coincidence with experimental data. The decrease in charge carriers mobility corresponds to a decrease in the drift component of the current of charge carriers. At the same time, an increase in the intensity of radiation processing of materials leads to an increase in the intensity of diffusion, stimulated by radiation. In this situation, one can find an increase in the diffusion component of the current of charge carriers (see *Fig. 2*).

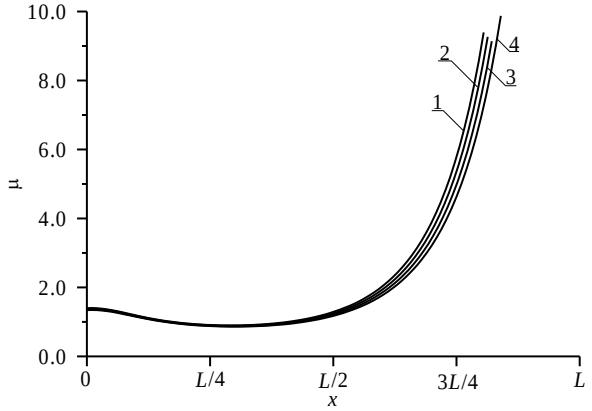


Fig. 3. Typical distributions of charge carrier mobility at different values of the dose of radiation processing in an implanted-junction rectifier.

An increase in the number of curves corresponds to an increase in the value of the dose.

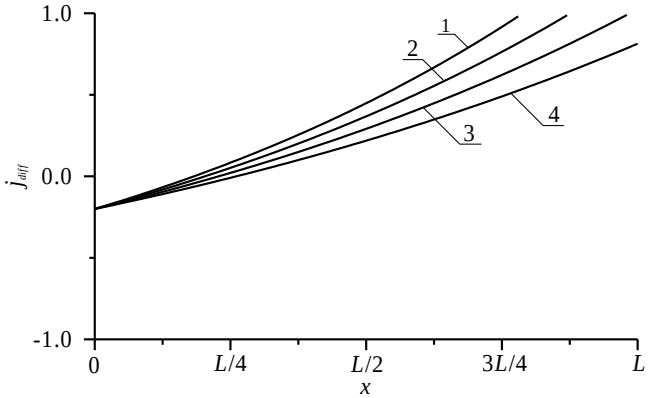


Fig. 4. Typical distributions of the diffusion component of current of charge carriers at different values of dose of radiation processing in implanted-junction rectifier.

An increase in the number of curves corresponds to a decrease in the value of the dose.

4 | Conclusion

We introduce a model for analysis of the influence of radiation processing on transport of charge carriers in p-n-junctions (we analyzed processes in implanted-junction rectifiers as an example of analysis), which were manufactured in multilayer structures. The model gives the possibility to take into account a larger quantity of factors in comparison with existing models in the literature. We also introduce an analytical approach to analyze the transport of charge carriers. The approach makes it possible to simultaneously take into account changes in the properties of materials in space and time, as well as the nonlinearity of the considered transport.

Authors' Contributions

All aspects of the research and manuscript preparation were carried out by the author. The author has read and approved the final version of the manuscript.

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All data supporting the reported findings in this research paper are provided within the manuscript.

Conflict of Interest

The author declares that they do not have any conflict of interest.

Consent for Publication

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Ethics Approval and Consent to Participate

This article does not contain any studies with human participants performed by the author.

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