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# A Survey of The Centroids of Fuzzy Numbers and Applications

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## Abstract

This paper presents a thorough review of the various methods for determining the centroids of fuzzy numbers, highlighting their significance in fuzzy set theory and decision-making processes. Starting with the foundational concepts of fuzzy sets and fuzzy numbers, including triangular and trapezoidal forms, the study critically examines existing centroid calculation formulas, addressing their advantages and limitations. The review identifies common errors in previous approaches, emphasizing the necessity for accurate and consistent centroid formulae. Furthermore, the paper explores multiple applications of centroid-based fuzzy number ranking, notably in decision-making and Multi-Criteria Decision Making (MCDM). It demonstrates that precise centroid computation is essential for effective fuzzy number comparison and ranking, ultimately enhancing the reliability of fuzzy logic applications in engineering, management, and applied sciences.

**Keywords:** Fuzzy numbers, Fuzzy set theory, Decision-making, Multi-criteria decision making, Fuzzy logic applications.

## 1 | Introduction

In 1965, Zadeh [1] established the theory of the fuzzy set. Atanassov [2] expanded Zadeh's [1] Fuzzy Sets in 1986; it is called the Theory of Intuitionistic Fuzzy Sets (IFSs). The fuzzy set shows uncertainty and instability. In fuzzy theory, uncertainty can be significantly reduced [1], and in the supply chain, underground mining, weaponry, and other items are used [3–10]. The concept of defuzzification in fuzzy theory means that a real number is attributed to a fuzzy number. Therefore, in defuzzification, there is a one-to-one correspondence between a fuzzy set and a real set. It is clear that in defuzzification, much information about the fuzzy set disappears [11–14].

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One of the special characteristics that is obtained in defuzzification is the centroids. The centroids in the fuzzy number ranking are very important because the inertia centroids lead to incorrect rankings and wrong applications. In the following, this paper first introduces the fuzzy sets [15]. In the next section, we describe a variety of ranking methods, and finally, we will focus on applications of the centroids.

## 2 | Fuzzy Set

A fuzzy set of elements in the reference set that have no distinct boundaries is fuzzy and confusing. In order to express the basics and principles of fuzzy theory, Zadeh [1] goes to the sets. A fuzzy set $\tilde{A}$ of the universe of discourse X is defined $\tilde{A} = \{(x, \mu_{\tilde{A}}(x) | x \in X \}$. Each fuzzy set with a unique membership function is shown, which shows the membership degree of a fuzzy set in a set. Membership function is in the range [0,1] and is displayed as μ. The membership function is a fuzzy number of piecewise continuous functions that has the following conditions:

I. $\mu\,(X) = 0$ outside some interval $[a, b]$;

II. $\mu_A(X)$ is non-decreasing (monotonic increasing) on $[a, b]$ and non-increasing (monotonic decreasing) on $[c, d]$;

III. $\mu_A(X) = 1$ for each $x \in [b, c]$, where $a \le b \le c \le d$ are real numbers in the real line R.

IV. A fuzzy set of the universe of discourse X is said to be convex if and only if for all $x_1$ and $x_2$ in X there always exists:

$$\mu_A(\lambda x_1 + (1+\lambda)x_2) \ge \min(\mu_{\tilde{A}}(x_1) + \mu_{\tilde{A}}(x_2)).$$

The most representative of the fuzzy numbers are trapezoidal and triangular fuzzy numbers, whose membership function is as follows:

$$f_{\tilde{A}}(x) = \begin{cases} f_{\tilde{A}}{}^{L}(x), & a \le x \le b, \\ \omega, & b \le x \le c, \\ f_{\tilde{A}}{}^{R}(x), & c \le x \le d, \\ o, & \text{otherwise.} \end{cases} \quad (1)$$

Also, for trapezoidal fuzzy numbers, the membership function is as follows:

$$f_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}\omega, & a \le x \le b, \\ \omega, & b \le x \le c, \\ \dfrac{d-x}{d-c}\omega, & c \le x \le d, \\ o, & \text{otherwise.} \end{cases} \quad (2)$$

See *(Fig. 1)*.

Since $f_{\tilde{A}}{}^{L}(x)$ and $f_{\tilde{A}}{}^{R}(x)$ both are strictly monotonic and continuous functions, their inverse functions exist and should also be continuous and purely monotonic. Let $g_{\tilde{A}}{}^{L} : [0, \omega] \to [c, d]$ and $g_{\tilde{A}}{}^{R} : [0, \omega] \to [c, d]$ be the inverse function of $f_{\tilde{A}}{}^{L}(x)$ and $f_{\tilde{A}}{}^{R}(x)$. This is the inverse of trapezoidal fuzzy numbers:

$$g_{\tilde{A}}{}^{L}(y) = a + (b-a)y / \omega, \qquad 0 \le y \le \omega.$$

$$g_{\tilde{A}}^{R} = d - (d-c)y/\omega, \quad 0 \le y \le \omega. \tag{3}$$

which are shown in *Fig. 2*.

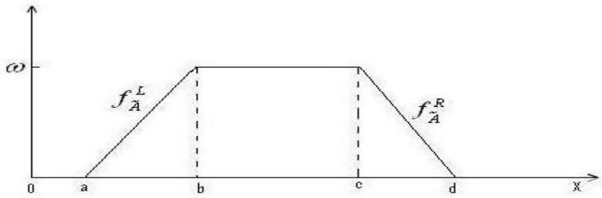


**Fig. 1. Membership function of trapezoidal fuzzy numbers.**

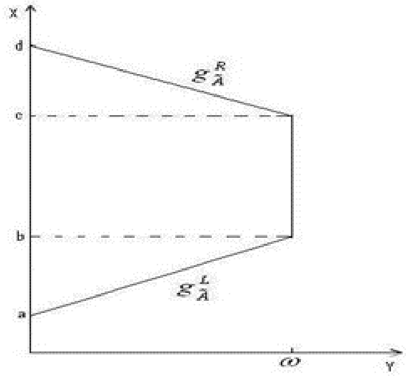


**Fig. 2. Exploring the inverse of trapezoidal fuzzy numbers: A mathematical approach to uncertainty modeling.**

Chanas et al. [16] defined eight relationships on a fuzzy set and proved some of them, and showed that four of them are very strong. One of the most important properties is that of s-transitiveness, which can be used in decision-making [17].

# 2 | Centroids of Fuzzy Numbers

Wang et al. [15] found that the centroid formulae provided by Cheng [18] were incorrect. To prevent misapplications, the authors showed the correct centroid formulae for ranking fuzzy numbers. After that, Shieh [19] extended the work of Wang et al. [15] to a more general case and presented a pair of correct centroid formulae for all fuzzy numbers. Suppose that the coordinate on the vertical axis is as essential as the coordinate on the horizontal axis. The method thus has consistent expression on both the horizontal and vertical axes and presents a significant improvement compared to previous research.

In another approach, Wang and Lee [20] pointed out problems in the Chu and Tsao [21] area ranking method and then proposed another ranking method. Fuzzy numbers are ranked based on the centroid values on the horizontal axis first, and then, if these values are equal. The review of index ranking methods has been reported in Ramli and Mohammed [22]. Different methods of ranking the fuzzy numbers centroid are considered, and the comparison shows that no single method in the concept of centroid is superior to all other methods, because each method has some advantages and disadvantages. Continue to read the described methods.

## 2.1 | Cheng Method

The centroid point $(\bar{x}_o, \bar{y}_o)$ for a fuzzy number $\tilde{A}$ is defined as

$$\bar{x}_0(\tilde{A}) = \frac{\int_a^b (xf_{\tilde{A}}^L(x))dx + \int_b^c (x)dx + \int_c^d xf(xf_{\tilde{A}}^R(x))dx}{\int_a^b (f_{\tilde{A}}^L(x))dx + \int_b^c dx + \int_c^d f_{\tilde{A}}^R(x))dx}. \quad (4)$$

$$\bar{y}_0(\tilde{A}) = \frac{\omega\left[\int_0^1 (yg_{\tilde{A}}^L(y))dy + \int_0^1 yg_{\tilde{A}}^R(y))dy\right]}{\int_0^1 (g_{\tilde{A}}^L(y))dy + \int_0^1 (g_{\tilde{A}}^R(y))dy}. \quad (5)$$

Especially for a trapezoidal fuzzy number $\tilde{A} = (a, b, c, d, \omega)$:

$$\bar{x}_0(\tilde{A}) = \frac{\int_a^b (xf_{\tilde{A}}^L(x))dx + \int_b^c (x)dx + \int_c^d xf(xf_{\tilde{A}}^R(x))dx}{\int_a^b (f_{\tilde{A}}^L(x))dx + \int_b^c dx + \int_c^d f_{\tilde{A}}^R(x))dx}. \quad (6)$$

$$\bar{y}_0(\tilde{A}) = \frac{\int_0^\omega yg_{\tilde{A}}^L(y))dy + \int_0^\omega yg_{\tilde{A}}^R(y))dy}{\int_0^\omega (g_{\tilde{A}}^L(y))dy + \int_0^\omega (g_{\tilde{A}}^R(y))dy}. \quad (7)$$

## 2.2 | Chu and Tsao Method

$$\bar{x}_0(\tilde{A}) = \frac{\omega(d^2 - 2c^2 + 2b^2 - a^2 + dc - ab) + 3(3c^2 - b^2)}{3\omega(d - c + b - a) + 6(c - b)}. \quad (8)$$

$$\bar{y}_0(\tilde{A}) = \omega\frac{1}{3}\left[1 + \frac{b + c}{a + b + c + d}\right]. \quad (9)$$

Especially for a trapezoidal fuzzy number $\tilde{A} = (a, b, c, d, \omega)$:

$$\bar{x}_0(\tilde{A}) = \frac{\omega(d^2 - 2c^2 + 2b^2 - a^2 + dc - ab) + 3(3c^2 - b^2)}{3\omega(d - c + b - a) + 6(c - b)}. \quad (10)$$

$$\bar{y}_0(\tilde{A}) = \omega\frac{1}{3}\left[1 + \frac{b + c}{a + b + c + d}\right]. \quad (11)$$

The main errors from *Formulae (4)*, *(8)* is remove $\omega$ and that both *Formulae (5)* and *(9)* take a positive sign for the second item in both numerator and denominator, which is a fundamental error and makes the formulae wrong for any $\omega$ values.

## 2.3 | Wang Method

The above formulae were also obtained by Pan and Yeh [23], [24]. However, it is found that the above *Formulae (4-9)* are incorrect despite the fact that *Formulae (4)* and *(8)* are consistent with Yager's ranking index [25], [26].

$$\bar{x}_0(\tilde{A}) = \frac{\int_{-\infty}^{+\infty} g(x)f_{\tilde{A}}(x)dx}{\int_{-\infty}^{+\infty} f_{\tilde{A}}(x)dx}. \quad (12)$$

with the weight function $g(x) = x$ and $x = 1$ and the same as Murakami et al.'s [27] ranking index for normal fuzzy sets.

$$\bar{x}_0(\tilde{A}) = \frac{\int_{-\infty}^{+\infty} x f_{\tilde{A}}(x)dx}{\int_{-\infty}^{+\infty} f_{\tilde{A}}(x)dx}.$$

The correct centroid formulae by wang should be as follows:

$$\bar{x}_0(\tilde{A}) = \frac{\int_{-\infty}^{+\infty} x f_{\tilde{A}}(x)dx}{\int_{-\infty}^{+\infty} f_{\tilde{A}}(x)dx} = \frac{\int_a^b x f_{\tilde{A}}^L(x) + \int_b^c (x\omega)dx + \int_c^d x f_{\tilde{A}}^R(x)dx}{\int_a^b f_{\tilde{A}}^L(x)dx + \int_b^c (\omega)dx + \int_c^d f_{\tilde{A}}^R(x)dx}. \tag{13}$$

$$\bar{y}_0(\tilde{A}) = \frac{\int_0^{\omega} y(g_{\tilde{A}}^R(y) - g_{\tilde{A}}^L(y))dy}{\int_0^{\omega} (g_{\tilde{A}}^R(y) - g_{\tilde{A}}^L(y))dy}. \tag{14}$$

Especially for a trapezoidal fuzzy number $\tilde{A} = (a,b,c,d,\omega)$:

$$\bar{x}_0(\tilde{A}) = \frac{1}{3}\left[a+b+c+d - \frac{dc-ab}{(d+c)-(a+b)}\right]. \tag{15}$$

$$y_0(\tilde{A}) = \omega\frac{1}{3}\left[1 + \frac{c-b}{(d+c)-(a+b)}\right]. \tag{16}$$

In order to overcome the shortcomings in Cheng's [18] distance method, Chu and Tsao's [21] formulae, and the new revised method by Wang and Lee [20], a sign function was introduced by Abbasbandy to compose an improvement strategy of Cheng's [18] distance. In this work, those methods are considered that utilize the centroid points. The improved method can effectively rank various fuzzy numbers and their images [28].

## 2.4 | Shieh Method

Shihe extended their work to a more general case and presented a pair of correct centroid formulae for all fuzzy numbers. The proposed formulae are developed based on the premise that the coordinate on the vertical axis is as essential as the coordinate on the horizontal axis and should be determined naturally, in the same way as the coordinate on the horizontal axis. A more restrictive condition is that the membership functions must be invertible. The proposed formulae are unlimited, so that they are available for all fuzzy numbers. This is a significant improvement on the formulae above. The formulas of this method are as follows:

$$\bar{x}_0(\tilde{A}) = \frac{\int_{-\infty}^{+\infty} x f(x)dx}{\int_{-\infty}^{+\infty} f(x)dx}.$$

$$\bar{y}_0(\tilde{A}) = \frac{\int_0^{\omega} \alpha |f^{\alpha}| d\alpha}{\int_0^{\omega} |f^{\alpha}| d\alpha}, \tag{17}$$

Where f is a fuzzy number with $\sup f(x) = \omega, \quad x \in R$ and $|f^{\alpha}|$ is the length of the $\alpha$ - cut $f^{\alpha}$, $0 < \alpha \le 1$. For the fuzzy numbers, the same formulas of the method Wang are obtained [19].

### 2.5 | Centroid of an Intuitionistic Fuzzy Number

Varghese and Kuriakose [29] introduced a formula for finding the centroid from Intuitionistic Fuzzy Number (IFN) and also proved that this is a generalization of the formula in the literature. This formula Triangular Intuitionistic Fuzzy Number (TIFN) is as follows:

The centroid of TIFN $\{(a,b,c),(e,b,g)\}$ is:

$$\frac{1}{3}\left[\frac{(g-e)(b-2g-2e)+(c-a)(a+b+c)+3(g^2-e^2)}{g-e+c-a}\right]. \tag{18}$$

## 3 | Centroids Applications

### 3.1 | Ranking Fuzzy Number

Ranking of fuzzy numbers plays a vital role in real-time programs, especially in decision-making. Many of the ranking methods for fuzzy numbers are proposed as follows:

Cheng [18] presented a ranking function $R(\tilde{A})=\sqrt{\bar{x}_o{}^2+\bar{y}_o{}^2}$ that is the distance from the centroid point to the origin. The larger the value of $R(\tilde{A})$, the better the ranking of $\tilde{A}$. Cheng's [18] method cannot rank crisp fuzzy numbers because the denominator becomes zero for crisp fuzzy numbers; hence, his centroid formulae are undefined for crisp numbers [19]. Chu and Tsao [21] showed a method for ranking fuzzy numbers based on a ranking function $R(A)=\bar{x}_o\bar{y}_o$, which is the area between the centroid point $(\bar{x}_o,\bar{y}_o)$ of each fuzzy number $\tilde{A}=(a,b,c,d,\omega)$ and the original point. Asady [30] by minimizing the distance between two fuzzy numbers, the closest point in terms of the fuzzy number was obtained, and given the rank of the nearest point, a method for fuzzy numbers was developed.

Parandin and Fariborz [31] presented their paper with a simple method using distance The defect in the Cheng [18] method was proposed. It can compare triangles and non-triangles of triangular and fuzzy triangles. This can easily and in many cases provide a satisfactory solution, ranking them. In 2008, Saneifard and Nahid [32] developed a method for ranking fuzzy numbers based on a positive and negative solution, and generalized it to integer numbers.

Saneifard [33] in his research proposes a modified new method to rank L-R fuzzy numbers. The modified method uses a defuzzification of parametrically represented fuzzy numbers that have been studied in [34]. This parameterized defuzzification can be used as a crisp approximation with respect to a fuzzy quantity. He used this defuzzification for ordering fuzzy numbers. The modified method can effectively rank various fuzzy numbers and their images and overcome the shortcomings of the previous techniques [33]. Kumar et al. [35], using the integral value approach of Liou and Wang [36], developed a ranking formula for comparing the exponential fuzzy numbers, which depends on the height of the fuzzy number. Also, it is proven that the ranking function for exponential fuzzy numbers is not linear.

Rao and Shanka [37] offer a new one, the fuzzy number ranking method, which uses the Centroids range and the optimism indicator to represent the decision, an optimistic attitude, and also an indicator of the manner in which the decision-maker is neutral. This method ranks different types of fuzzy numbers that include the normal, triangular, and triangular fuzzy numbers, with crisp numbers of the property that clear numbers must be considered in some instances of fuzzy numbers [37]. In order to rank all fuzzy numbers, Ezzati et al. [38] modify the method of “a new approach for ranking of trapezoidal fuzzy numbers” by Abbasbandy and Hajjari [39]. The proposed method is used for ranking symmetric fuzzy numbers.

In Yong a and Qi [40], a new method for ranking fuzzy numbers was proposed based on the COG point. They showed that the proposed method can overcome the drawbacks of other existing centroid-index ranking methods. Ganesh and Jayakumar [41] proposed a new approach to the fuzzy ranking of generalized trapezoidal fuzzy numbers based on the radius of gyration point of centroids. The main advantage of the proposed approach is that it provides the correct ordering of generalized and normal trapezoidal fuzzy numbers. Also, in the same year, Dhanasekar [42] ranked trapezoidal fuzzy numbers by using the Haar wavelet. Barkhordari Ahmadi [43] showed a novel approach for ranking fuzzy numbers based on the angle measure. Several left and right spreads at each chosen level of fuzzy numbers are used to determine center of mass points (CMPs), and then, the angles between the CMPs and the horizontal axis are calculated. The total angle is determined by averaging the computed angles.

Based on revising the ranking method proposed by Ezzati et al. [38]. Ezzati et al. [44] proposed a new technique. They presented a new approach for the ranking of all trapezoidal fuzzy numbers, and consequently, they overcome the drawbacks of some related methods. All properties were studied in detail and finally by using some numerical methods. In 2016, Lotfi et al. [45] proposed a new approach for constructing a preference fuzzy distance measure. For this purpose, they proposed a new and effective preference ordering based on Abbasbandy and Hajjari's [39] approach.

## 3.2 | Find an Unknown Fuzzy Number by the Centroid and the Left /Right Spread

Hadi [46] showed by the formula below how to obtain an unknown fuzzy number with its center of centroid and the left /right spread, also indicating that the system is under a linear system. Suppose $\tilde{A}=(a,b,c,d)$ is an unknown trapezoidal fuzzy number with the centroid point $(\bar{x}_o,\bar{y}_o)$, the left spread $(L)$, and the right spread $(R)$. The fuzzy number is obtained as follows [46]:

$$\begin{cases} b-a=L, \\ d-c=R, \\ \frac{1}{3}[a+b+c+d-\frac{cd-ad}{c+d-a-b}]=\bar{x}_o, \\ \frac{1}{3}[1+\frac{c-b}{c+d-a-b}]=\bar{y}_o. \end{cases} \quad (19)$$

## 3.4 | Centroid of Fuzzy Number and Multi-Criteria Decision Making

Hadi [46] in his paper proposed a new Fuzzy Multi-Criteria Decision Making (FMCDM) approach based on the centroid of fuzzy numbers for ranking of alternatives. The FMCDM approach allows Decision-Makers (DMs) to evaluate alternatives using linguistic terms such as very high, high, slightly high, medium, somewhat low, low, very low, or none rather than precise numerical values, allows them to express their opinions independently, and also provides an algorithm to aggregate the assessments of alternatives—the proposed a fuzzy MCDM method based on the centroid of a fuzzy number. Suppose a fuzzy MCDM problem has n alternatives $A_1,...,A_n$ and m decision criteria $C_1,...,C_m$, and let $\tilde{X}=(\tilde{x}_{ij})$ be a fuzzy decision matrix, $\tilde{w}=(\tilde{w}_1,...\tilde{w}_m)$ be fuzzy weight, $\tilde{x}_{ij}=(\alpha_{ij},\beta_{ij})$ and $\tilde{w}_j=(\gamma_j,\delta_j)$ be the centroid of $\tilde{x}_{ij}$, $\tilde{w}_j$ respectively. In short, the fuzzy MCDM method is based on fuzzy centralization numbers, which can be as follows:

I. Determine the centroid of each $\tilde{x}_{ij}$ and $\tilde{w}_j$ by *Eq. (15).*

II. Calculate the centroid of the fuzzy weighted average of each alternative by the following *Eqs. (21-22)*:

$$\bar{\theta}_i=(\eta_i,\lambda_i)=\frac{\sum_{j=1}^{m}\left[\alpha_{ij},\beta_{ij}).(\gamma_j,\delta_j\right]}{\sum_{j=1}^{m}(\gamma_j,\delta_j)}=\frac{(\mu,\nu)}{(\sum_{j=1}^{m}\gamma_j)^2+(\sum_{j=1}^{m}\delta_j)^2}. \quad (20)$$

$$\mu = \sum_{j=1}^{m}\left[\alpha_{ij}\gamma_j - \beta_{ij}\delta_j\right]\sum_{j=1}^{m}\gamma_j + \sum_{j-1}^{m}\left[\alpha_{ij}\delta_j + \beta_{ij}\gamma_j\right]\sum_{j=1}^{m}\delta_j. \tag{21}$$

$$\nu = \sum_{j=1}^{m}\left[\alpha_{ij}\delta_j + \beta_{ij}\gamma_j\right]\sum_{j=1}^{m}\gamma_j - \sum_{j=1}^{m}\left[\alpha_{ij}\gamma_j - \beta_{ij}\delta_j\right]\sum_{j=1}^{m}\delta_j. \tag{22}$$

I. Generate the overall score of alternatives $A_i$ by $s_i = \sqrt{\eta_i^2 + \lambda_i^2}$.

II. Rank and prioritize alternatives according to their overall scores: A high score means high priority [47].

# 4 | Conclusion

This review underscored the critical importance of accurate centroid calculation in the analysis and application of fuzzy numbers. It highlighted that the adoption of standardized, error-free formulas is essential to ensure reliable fuzzy number ranking and decision-making. The examination of various methodologies revealed that, despite the lack of a universally superior approach, consistent implementation of precise formulas—particularly for trapezoidal fuzzy numbers—is vital for maintaining accuracy and comparability. Moreover, the widespread applications in decision-making, optimization, and other fields demonstrated that the integrity of centroid computations directly affects system performance and outcome validity. Moving forward, further research should aim to refine existing methods and develop innovative approaches, thereby strengthening the theoretical foundation and practical utility of fuzzy set theory across diverse disciplines.

# Conflict of Interest Disclosure

All authors certify that they have no affiliations with or involvement in any organization or entity with any financial or non-financial interest in the subject matter discussed in this manuscript.

# Data Availability Statement

The datasets used and/or analyzed during the current study are not publicly available due to [reason if applicable] but can be made available by the corresponding author when scientifically justified.

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# References

[1] Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, *8*(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X

[2] Atanassov, K. (1986). Intuitionistic fuzzy sets. fuzzy sets and systems. *Fuzzy sets and systems*, *20*(1), 87–96. http://dx.doi.org/10.1016/S0165-0114(86)80034-3

[3] Chu, T. C., & Lin, Y. C. (2003). A fuzzy TOPSIS method for robot selection. *The international journal of advanced manufacturing technology*, *21*, 284–290. https://doi.org/10.1007/s001700300033

[4] Chou, S. Y., & Chang, Y. H. (2008). A decision support system for supplier selection based on a strategy-aligned fuzzy SMART approach. *Expert systems with applications*, *34*(4), 2241–2253. https://doi.org/10.1016/j.eswa.2007.03.001

[5] Chou, S. Y., Chang, Y. H., & Shen, C. Y. (2008). A fuzzy simple additive weighting system under group decision-making for facility location selection with objective/subjective attributes. *European journal of operational research*, *189*(1), 132–145. https://doi.org/10.1016/j.ejor.2007.05.006

[6] Tseng, M. L. (2010). An assessment of cause and effect decision-making model for firm environmental knowledge management capacities in uncertainty. *Environmental monitoring and assessment*, *161*, 549–564. https://doi.org/10.1007/s10661-009-0767-2

[7] Tseng, M. L. (2011). Using a hybrid MCDM model to evaluate firm environmental knowledge management in uncertainty. *Applied soft computing*, *11*(1), 1340–1352. https://doi.org/10.1016/j.asoc.2010.04.006

[8] Tseng, M. L. (2011). Green supply chain management with linguistic preferences and incomplete information. *Applied soft computing*, *11*(8), 4894–4903. https://doi.org/10.1016/j.asoc.2011.06.010

[9] Azadeh, A., Osanloo, M., & Ataei, M. (2010). A new approach to mining method selection based on modifying the Nicholas technique. *Applied soft computing*, *10*(4), 1040–1061. https://doi.org/10.1016/j.asoc.2009.09.002

[10] Dağdeviren, M., Yavuz, S., & Kılınç, N. (2009). Weapon selection using the AHP and TOPSIS methods under fuzzy environment. *Expert systems with applications*, *36*(4), 8143–8151. https://doi.org/10.1016/j.eswa.2008.10.016

[11] Filev, D. P., & Yager, R. R. (1991). A generalized defuzzification method via BAD distributions. *International journal of intelligent systems*, *6*(7), 687–697. https://doi.org/10.1002/int.4550060702

[12] Kandel, A. (1986). *Fuzzy mathematical techniques with applications*. Addison-Wesley Longman Publishing Co. https://doi.org/10.5555/6765

[13] Yager, R. R., & Filev, D. P. (1993). Slide: A simple adaptive defuzzification method. *IEEE transactions on fuzzy systems*, *1*(1), 69. https://doi.org/10.1109/TFUZZ.1993.390286

[14] Abbasbandy, S., & Asady, B. (2004). The nearest trapezoidal fuzzy number to a fuzzy quantity. *Applied mathematics and computation*, *156*(2), 381–386. https://doi.org/10.1016/j.amc.2003.07.025

[15] Wang, Y. M., Yang, J. B., Xu, D. L., & Chin, K. S. (2006). On the centroids of fuzzy numbers. *Fuzzy sets and systems*, *157*(7), 919–926. https://doi.org/10.1016/j.fss.2005.11.006

[16] Chanas, S., Delgado, M., Verdegay, J. L., & Vila, M. A. (1993). Ranking fuzzy interval numbers in the setting of random sets. *Information sciences*, *69*(3), 201–217. https://doi.org/10.1016/0020-0255(93)90120-B

[17] Orlovsky, S. (1993). Decision-making with a fuzzy preference relation. In *Readings in fuzzy sets for intelligent systems* (pp. 717–723). Elsevier. https://doi.org/10.1016/B978-1-4832-1450-4.50077-8

[18] Cheng, C. H. (1998). A new approach for ranking fuzzy numbers by distance method. *Fuzzy sets and systems*, *95*(3), 307–317. https://doi.org/10.1016/S0165-0114(96)00272-2

[19] Shieh, B. S. (2007). An approach to centroids of fuzzy numbers. *International journal of fuzzy systems*, *9*(1), 51–54. https://www.researchgate.net/publication/238068920_An_Approach_to_Centroids_of_Fuzzy_Numbers

[20] Wang, Y. J., & Lee, H. S. (2008). The revised method of ranking fuzzy numbers with an area between the centroid and original points. *Computers & mathematics with applications*, *55*(9), 2033–2042. https://doi.org/10.1016/j.camwa.2007.07.015

[21] Chu, T. C., & Tsao, C. T. (2002). Ranking fuzzy numbers with an area between the centroid point and original point. *Computers & mathematics with applications*, *43*(1–2), 111–117. https://doi.org/10.1016/S0898-1221(01)00277-2

[1] [22] Ramli, N., & Mohamad, D. (2009). A comparative analysis of centroid methods in ranking fuzzy numbers. *European journal of scientific research*, *28*(3), 492–501. https://doi.org/10.5923/j.ijcem.20160502.02

[23] Pan, H., & Yeh, C. H. (2003). Fuzzy project scheduling. *The 12th ieee international conference on fuzzy systems, 2003. fuzz'03.* (Vol. 1, pp. 755–760). IEEE. https://doi.org/10.1109/FUZZ.2003.1209458

[24] Pan, H., & Yeh, C. H. (2003). A metaheuristic approach to fuzzy project scheduling. *International conference on knowledge-based and intelligent information and engineering systems* (pp. 1081–1087). Springer. http://dx.doi.org/10.1007/978-3-540-45224-9_145

[25] Yager, R. R. (1979). Ranking fuzzy subsets over the unit interval. *1978 IEEE conference on decision and control including the 17th symposium on adaptive processes* (pp. 1435–1437). IEEE. https://ieeexplore.ieee.org/abstract/document/4046341/

[26] Yager, R. R. (1981). A procedure for ordering fuzzy subsets of the unit interval. *Information sciences*, *24*(2), 143–161. https://doi.org/10.1016/0020-0255(81)90017-7

[27] Murakami, S., & Maeda, M. (1984). Fuzzy decision analysis in the development of centralized regional energy control systems. *Energy development, (United States)*, *6*(4). https://www.osti.gov/biblio/5035693

[28] Abbasbandy, S., & Hajjari, T. (2011). An improvement in centroid point method for ranking of fuzzy numbers, *20*(78), 109–117. https://www.sid.ir/FileServer/JE/959201178-201 \

[29] Varghese, A., & Kuriakose, S. (2012). Centroid of an intuitionistic fuzzy number. *Notes on intuitionistic fuzzy sets*, *18*(1), 19–24. https://ifigenia.org/images/archive/1/1b/20120721115607!NIFS-18-1-19-24.pdf

[30] Asady, B., & Zendehnam, A. (2007). Ranking fuzzy numbers by distance minimization. *Applied mathematical modelling*, *31*(11), 2589–2598. https://doi.org/10.1016/j.apm.2006.10.018

[31] Parandin, N., & Fariborzi, A. M. A. (2008). Ranking of fuzzy numbers by distance method, *5*(19), 47–55. https://www.sid.ir/FileServer/JE/134520081905

[32] Saneifard, R., & Nahid, S. (2013). A new parametric method for ranking fuzzy numbers based on positive and negative ideal solutions, *5*(2), 119–128. https://sid.ir/paper/231733/en

[33] Saneifard, R. (2009). Ranking LR fuzzy numbers with weighted averaging based on levels, *1*(2), 163–173. https://doi.org/1016220090206.pdf

[34] Nasibov, E. N., & Mert, A. (2007). On methods of defuzzification of parametrically represented fuzzy numbers. *Automatic control and computer sciences*, *41*, 265–273. https://doi.org/10.3103/S0146411607050057

[35] Kumar, A., Singh, P., & Kaur, A. (2010). Ranking of generalized exponential fuzzy numbers using integral value approach. *International journal of advances in soft computing and its applications*, *2*(2), 1–10. https://www.academia.edu/download/77445202/vol.2.2.5.July.10.pdf

[36] Liou, T. S., & Wang, M. J. J. (1992). Ranking fuzzy numbers with integral value. *Fuzzy sets and systems*, *50*(3), 247–255. https://doi.org/10.1016/0165-0114(92)90223-Q

[37] Rao, P. P. B., & Shankar, N. R. (2011). Ranking fuzzy numbers with a distance method using circumcenter of centroids and an index of modality. *Advances in fuzzy systems*, *2011*(1), 178308. https://doi.org/10.1155/2011/178308

[38] Ezzati, R., Allahviranloo, T., Khezerloo, S., & Khezerloo, M. (2012). An approach for ranking of fuzzy numbers. *Expert systems with applications*, *39*(1), 690–695. https://doi.org/10.1016/j.eswa.2011.07.060

[39] Abbasbandy, S., & Hajjari, T. (2009). A new approach for ranking of trapezoidal fuzzy numbers. *Computers & mathematics with applications*, *57*(3), 413–419. https://doi.org/10.1016/j.camwa.2008.10.090

[40] Yong, D., & Qi, L. (2005). A TOPSIS-based centroid--index ranking method of fuzzy numbers and its application in decision-making. *Cybernetics and systems: An international journal*, *36*(6), 581–595. https://doi.org/10.1080/01969720590961727

[41] Ganesh, A. H., & Jayakumar, S. (2014). Ranking of fuzzy numbers using radius of gyration of centroids. *International journal of basic and applied sciences*, *3*(1), 17. http://dx.doi.org/10.14419/ijbas.v3i1.1477

[42] Dhanasekar, S., Hariharan, S., & Sekar, P. (2014). Ranking of generalized trapezoidal fuzzy numbers using Haar wavelet. *Applied mathematical sciences*, *8*(160), 7951–7958. http://dx.doi.org/10.12988/ams.2014.410798

[43] Barkhordari, A. M. (2014). Ranking of fuzzy numbers based on angle measure, *3*(1), 259–265. http://dx.doi.org/1046820140106.pdf

[44] Ezzati, R., Khezerloo, S., & Ziari, S. (2015). Application of parametric form for ranking of fuzzy numbers. *Iranian journal of fuzzy systems*, *12*(1), 59–74. https://ijfs.usb.ac.ir/article_1842_27cb3eea8770ed442d2b473b328b2217.pdf

[45] Lotfi, M., Salahshour, S., Esfahani, F. N., & Jafarnejad, A. (2016). A new approach for ranking fuzzy numbers. *Journal of fuzzy set valued analysis*, *2016*, 102–109. http://dx.doi.org/10.5899/2016/jfsva-00277

[46] Hadi-Vencheh, A., & Mokhtarian, M. N. (2011). A new fuzzy MCDM approach based on centroid of fuzzy numbers. *Expert systems with applications*, *38*(5), 5226–5230. https://doi.org/10.1016/j.eswa.2010.10.036

[47] Hadi-Vencheh, A., & Allame, M. (2010). On the relation between a fuzzy number and its centroid. *Computers & mathematics with applications*, *59*(11), 3578–3582. https://doi.org/10.1016/j.camwa.2010.03.051